Bachelor of Science (B.Sc.) Semester–I (C.B.S.) Examination
MATHEMATICS (M_2-Calculus)
Compulsory Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve **all** the five questions.
(2) All questions carry equal marks.
(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using $\varepsilon-\delta$ definition, show that :

$$\lim_{x\to 3}\left(2x^2+x\right)=21$$ 6

(B) Show that the function :

$$f(x)=\begin{cases}\dfrac{|x-2|}{x-2} & , \quad x\neq 2\\ 1 & , \quad x=2\end{cases}$$

has a discontinuity of the first kind at x = 2. 6

OR

(C) Show that the function f defined by

$$f(x)=\begin{cases}2+x & \text{if} \quad x\geq 0\\ 2-x & \text{if} \quad x<0\end{cases}$$

is continuous at x = 0 but not derivable at x = 0. 6

(D) If $y=(\sin^{-1}x)^2$, then prove that

$(1-x^2)y_2-xy_1-2=0.$

Differentiate the above equation n times with respect to x and show that :

$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0.$ 6

UNIT—II

2. (A) Expand $\tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$. 6

(B) If a curve is defined by the equation $x = f(t)$ and $y = \phi(t)$, then prove that radius of curvature

is $\rho = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}}$

Where (.) denotes the derivative with respect to t. 6

OR

(C) Find the asymptotes of the curve $x^2y + xy^2 + xy + y^2 + 3x = 0$. 6

(D) Evaluate : $\lim_{x \to 0} (\cot x)^{\frac{1}{\log x}}$ 6

UNIT—III

3. (A) If $x^x y^y z^z = C$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -\left(x \log ex\right)^{-1}$. 6

(B) If $u = x^2 - y^2$, $x = 2r - 3s + 4$,
$y = -r + 8s - 5$ then find

$\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$. 6

OR

(C) If $Z = f(x,y)$ is a homogeneous function of degree n, then prove that

(i) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ and

(ii) $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$. 6

(D) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta. \qquad 6$$

UNIT—IV

4. (A) Evaluate $\int_0^1 \frac{1 - 4x + 2x^2}{\sqrt{2x - x^2}}\,dx.$ 6

(B) Evaluate $\int \frac{1}{(x-1)\sqrt{x^2 + x + 1}}\,dx.$ 6

OR

(C) Prove that :

$$\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx.$$

Hence, if $I_n = \int_0^{p/4} \tan^n x\,dx$ then show that

$$I_n + I_{n-2} = \frac{1}{n-1}. \qquad 6$$

(D) Prove that $\int_0^{p/2} \log(\sin x)\,dx = -\frac{p}{2}\log 2.$ 6

Question—V

5. (A) Show that $f(x) = \begin{cases} (1+x)^{1/x} & , \ x \neq 0 \\ 1 & , \ x = 0 \end{cases}$

has removable discontinuity at $x = 0$. 1½

(B) If $y = a^x$, then prove that $y_n = a^x (\log a)^n$. 1½

(C) Expand $f(x) = e^x$ by Maclaurin's theorem. 1½

(D) Determine $\lim_{x \to 0} (x.\log x)$ 1½

(E) If $z = \log\left(\frac{x^2 + y^2}{x + y}\right)$, Prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$. 1½

(F) Find $\frac{dz}{dt}$, when $z = xy^2 + x^2y$, where $x = at^2$, $y = 2at$. 1½

(G) Show that $\int_0^{\pi/2} \log \tan x \, dx = 0$. 1½

(H) Evaluate $\int \frac{1}{\sqrt{x^2 + 2x + 3}} dx$. 1½